## X-ray analysis

## 1. Basic crystallography

2. Basic diffraction physics
3. Experimental methods

## Introduction Noble prizes associated with X-ray diffraction

- 1901 W. C. Roentgen (Physics) for the discovery of X-rays.
- 1914 M. von Laue (Physics) for X-ray diffraction from crystals.
- 1915 W. H. and W. L. Bragg (Physics) for structure derived from X-ray diffraction.
- 1917 C. G. Barkla (Physics) for characteristic radiation of elements.
- 1924 K. M. G. Siegbahn (Physics) for X-ray spectroscopy.
- 1927 A. H. Compton (Physics) for scattering of X-rays by electrons.
- 1936 P. Debye (Chemistry) for diffraction of X-rays and electrons in gases.

- 1962 M. Perutz and J. Kendrew (Chemistry) for the structure of hemoglobin.
- 1962 J. Watson, M. Wilkins, and F. Crick (Medicine) for the structure of DNA.
- 1979 A. Cormack and G. Newbold Hounsfield (Medicine) for computed axial tomography.
- 1981 K. M. Siegbahn (Physics) for high resolution electron spectroscopy.
- 1985 H. Hauptman and J. Karle (Chemistry) for direct methods to determine structures.
- 1988 J. Deisenhofer, R. Huber, and H. Michel (Chemistry) for the structures of proteins that are crucial to photosynthesis.


## Introduction What's the result of X-ray analyses?

Crystal data
Formula sum
Crystal system
Space group
Unit cell dimensions
Z

Atomic coordinates

| Atom | Ox. | Wyck. | $\mathbf{x}$ | $\mathbf{y}$ | $\mathbf{z}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Mg1 | +2 | $4 a$ | 0.00000 | 0.00000 | 0.00000 |
| Mg2 | +2 | $4 c$ | $0.00995(600)$ | $0.27734(600)$ | 0.75000 |
| Si1 | +4 | $4 c$ | $0.07373(500)$ | $0.4043(50)$ | 0.25000 |
| O1 | -2 | $4 c$ | $0.23242(1000)$ | $0.0918(100)$ | 0.75000 |
| O2 | -2 | $4 c$ | $0.2793(100)$ | $0.05078(1000)$ | 0.25000 |
| O3 | -2 | $8 d$ | $0.22266(1000)$ | $0.33594(1000)$ | $0.46289(1000)$ |

- Structure
- Chemical information (bonding, composition)
- Real structure (defects)
orthorhombic
Pbn m (no. 62)
$a=4.75(2) \AA, b=10.25(4) \AA, c=6.00(2) \AA$
4


## 1. Basic crystallography Lattice, motif and structure

Example:
structure and lattice in 2D

- Lattice
- pattern of points

- no chemical information, mathematical description
- no atoms, but points and lattice vectors ( $\mathbf{a}, \mathbf{b}, \mathbf{c}, \alpha, \beta, \gamma$ ), unit cell
- Motif (characteristic structural feature, atom, group of atoms...)
- Structure = Lattice + Motif
- contains chemical information (e. g. environment, bond length...)
- describes the arrangement of atoms (symmetry of the crystal)


## 1. Basic crystallography Unit cell: interconnection of lattice and structure

## Definition:

Unit cell = parallel sided region of the lattice from which the entire crystal can be constructed by purely translational displacements

- contents of unit cell represents chemical composition (multiples of chemical formula)
- primitive cell: simplest cell, contains one lattice point
- centered cell: more than one point inside unit cell


Conventions:

1. Cell edges should coincide with symmetry axes or reflection planes
2. The smallest possible cell which fulfills 1 . should be chosen

# 1. Basic crystallography Unit cell: exercise 

Determine the primitive unit cell and one example for a centered setting

$$
\begin{array}{ccc}
0 & 0 \\
0
\end{array}
$$



Forbidden symmetry produced by superposition

## 1. Basic crystallography <br> Unit cells and crystal system

- millions of periodic structures but 7 types of primitive cells (crystal systems)
- crystal system = particular restriction concerning the unit cell
- crystal system = unit cell with characteristic symmetry elements (later)

| Crystal system | Restrictions axes | Restrictions angles |
| :--- | :---: | :---: |
| Triclinic | - | - |
| Monoclinic | - | $\alpha=\gamma=90^{\circ}$ |
| Orthorhombic | - | $\alpha=\beta=\gamma=90^{\circ}$ |
| Tetragonal | $\mathrm{a}=\mathrm{b}$ | $\alpha=\beta=\gamma=90^{\circ}$ |
| Trigonal | $\mathrm{a}=\mathrm{b}$ | $\alpha=\beta=90^{\circ}, \gamma=120^{\circ}$ |
| Hexagonal | $\mathrm{a}=\mathrm{b}$ | $\alpha=\beta=90^{\circ}, \gamma=120^{\circ}$ |
| Cubic | $\mathrm{a}=\mathrm{b}=\mathrm{c}$ | $\alpha=\beta=\gamma=90^{\circ}$ |

1. Basic crystallography

## Indexation of directions in direct space

"[uvw] = [110]"

## Procedure in three steps



1. Select 000
2. Mark position of second point


Convention: right-handed coordinate system

- middle finger: a
- thumb: b
- forefinger: c

1. Basic crystallography Fractional coordinates (position of the atoms)


- possible values for $x, y, z:[0 ; 1]$, atoms are multiplied by translations
- atoms are generated by symmetry elements (later)
- Example: Sphalerite (ZnS)
- Equivalent points are represented by one triplet only
- equivalent by translation
- equivalent by other symmetry elements (later)


## 1. Basic crystallography Centered unit cells- Bravais-type

The Bravais-types P, F, I, C, A, B, R denote centerings of the unit cells

- Centering of unit cell = Translation
- 7 crystal systems, 14 characteristic unit cells (i.e. Bravais-type)
- 7 types of centerings:
- $P$ : no centering ( $0,0,0$ )
- F: translation of each point by (1/2,1/2,0);(0,1/2,1/2);(1/2,0,1/2)
- I: translation of each point by $(1 / 2,1 / 2,1 / 2)$
- C: translation of each point by $(1 / 2,1 / 2,0)$
- A: translation of each point by $(0,1 / 2,1 / 2)$
- B: translation of each point by $(1 / 2,0,1 / 2)$
- R: translation of each point by ( $2 / 3,1 / 3,1 / 3$ );(1/3,2/3,2/3)
- All fractional coordinates are multiplied by centerings

(a)



# 1. Basic crystallography Bravais-type: example 

| Crystal data |  |
| :--- | :--- |
| Formula sum | NaCl |
| Crystal system | cubic |
| Space group | $\mathrm{Fm}-3 \mathrm{~m}$ (no. 225) |
| Unit cell dimensions | $a=5.6250(5) \AA$ |
| $Z$ | 4 |

Halite
Atomic coordinates

| Atom | Ox. | Wyck. | x | $\mathbf{y}$ | $\mathbf{z}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Na | +1 | $4 a$ | 0 | 0 | 0 |
| Cl | -1 | $4 b$ | $1 / 2$ | $1 / 2$ | $1 / 2$ |

# 1. Basic crystallography Wyckoff-notation and occupancy factor 

Crystal data
Formula sum
Crystal system
Space group
Unit cell dimensions
Z
$\mathrm{Cu}_{0.8} \mathrm{In}_{2.4} \mathrm{Se}_{4}$
tetragonal
I-4 2 m (no. 121)
$a=5.7539(3) \AA \quad c=11.519(1) \AA$
2

Atomic coordinates
Molecules

| Atom | Ox. | Wyck. | Occ. | $\mathbf{x}$ | $\mathbf{y}$ | z |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Cu1 | +1 | $2 a$ | 0.8 | 0 | 0 | 0 |
| $\ln 1$ | +3 | $4 d$ | 1.0 | 0 | $1 / 2$ | $1 / 4$ |
| $\ln 2$ | +3 | $2 b$ | 0.4 | 0 | 0 | $1 / 2$ |
| Se1 | -2 | $8 i$ | 1.0 | $1 / 4$ | $1 / 4$ | $1 / 8$ |

Occ. factors $\times$ Wyckoff number $=$ no. of atoms/unit cell

# 1. Basic crystallography Crystallographic symmetry elements 

SE in crystallography:

- Inversion
- Mirror
- Rotation axes: 1,2,3,4,6
- Translations


Question:
Why is e. g. 5 forbidden in crystal structures?


Coupling:

- both SE are applied in one step
- one of the two coupled SE is an intermedium


What about $\overline{\mathbf{2}}$ ?


1. Basic crystallography

Coupling- rotation and translation- Screw axes $X_{n}$
Strategy: X-fold rotation after translation of $n / X$ along the screw axis
$2_{1}$
$3_{1} 3_{2}$
$4_{1} 4_{3} 4_{2}$
$66_{5} 6_{2} 6_{4} 6_{3}$


# 1. Basic crystallography Coupling- mirror and translation - Gilde planes 

intermedium
real point

$a, b, c, n, d, e$
$a, b, c: 1 / 2 a, 1 / 2 b, 1 / 2 c$
$n: 1 / 2(a+b), 1 / 2(a+c), 1 / 2(b+c)$
d: $1 / 2 \mathrm{n}$

# 1. Basic crystallography Combination of mirror and rotation 

Combination:

- both SE are applied (two steps)
- no intermedium


2/m


3/m

## 1. Basic crystallography Directions of characteristic symmetry elements

| Crystal system | Characteristic SE | Char. direction / sequence |
| :--- | :---: | :---: |
| Triclinic | $1, \overline{1}$ | - |
| Monoclinic | 2 and/or m | b |
| Orthorhombic | $2(3 \times)$ and/or $\mathrm{m}(3 \times)$ | $\mathrm{a}, \mathrm{b}, \mathrm{c}$ |
| Trigonal | $3(1 \times)$ | $\mathrm{c}, \mathrm{a},[210]$ |
| Tetragonal | $4(1 \times)$ | $\mathrm{c}, \mathrm{a},[110]$ |
| Hexagonal | 6 | $\mathrm{c}, \mathrm{a},[210]$ |
| Cubic | $3(4 \times)$ | $[111], \mathrm{a},[110]$ |

What can we learn from this table?

- The knowledge of the characteristic SE allows to determine the crystal system
- Pseudometrics possible, e. g. monoclinic with $\beta=90^{\circ}$
- Systematization of SE in space group symbols


## Repetition Basic terms of 3D crystallography

- Lattice, motif and structure
- Unit cell (primitive and centered)
- Wyckoff notation, occupancy factor, composition of crystals
- Crystal system
- Bravais type
- Crystallographic SE

- Coupling of SE: rotoinversion, screw axes, glide planes
- Combination of SE
- Characteristic SE of the crystal casses


## 1. Basic crystallography Crystal class

32 Crystal classes: Combination of SE without translations

| Crystal system | Restriction | Crystal Class |
| :---: | :---: | :---: |
| Triclinic | - | 1, $\overline{1}$ |
| Monoclinic | $\alpha=\gamma=90^{\circ}$ | 2, m, 2/m |
| Orthorhombic | $\begin{gathered} \mathbf{a}=\mathbf{b} \\ \alpha=\beta=\gamma=90^{\circ} \end{gathered}$ | 222, mm2, 2/m 2/m 2/m |
| Trigonal | $\begin{gathered} \mathbf{a}=\mathbf{b} \\ \alpha=\beta=90^{\circ}, \gamma=120^{\circ} \end{gathered}$ | 3, $\overline{3}, 32,3 \mathrm{~m}, \overline{3} 2 / \mathrm{m}$ |
| Tetragonal | $\begin{gathered} \mathbf{a}=\mathbf{b} \\ \alpha=\beta=\gamma=90^{\circ} \end{gathered}$ | $\begin{gathered} 4, \overline{4}, 4 / \mathrm{m}, 422,4 \mathrm{~mm}, \overline{4} 2 \mathrm{~m} \\ 4 / \mathrm{m} 2 / \mathrm{m} \mathrm{2/m} \end{gathered}$ |
| Hexagonal | $\begin{gathered} \mathbf{a}=\mathbf{b} \\ \alpha=\beta=90^{\circ}, \gamma=120^{\circ} \end{gathered}$ | 6, $\overline{6}, 6 / \mathrm{m}, 622,6 \mathrm{~mm}, \overline{6} \mathrm{~m} 2$, $6 / \mathrm{m} \mathrm{2} / \mathrm{m} 2 / \mathrm{m}$ |
| Cubic | $\begin{gathered} \mathbf{a}=\mathbf{b}=\mathbf{c} \\ \alpha=\beta=\gamma=90^{\circ} \end{gathered}$ | $\begin{gathered} 23,2 / m \overline{3}, 432, \overline{4} 3 m \\ 4 / m \overline{3} 2 / m \end{gathered}$ |

The atoms ( $A, B$ and $C$ ) of a tetragonal structure are located on: $\mathbf{0}^{1 / 20}$ (A), $1 /{ }_{2}{ }^{1 / 20}$ (B) and 000 (C). Assume low symmetry

- Sketch the unit cell along [001]
- Determine the composition of a crystal with this structure
- Describe the environment of A, B and C

Repeat the exercise for a cubic structure!

## 1. Basic crystallography Crystal Classes- examples



| Crystal System | Charact. SE | Direction / Sequence |
| :--- | :---: | :---: |
| Triklin | $1, \overline{1}$ | - |
| Monoklin | 2 and/or $m$ | $b$ |
| Orthorhombisch | $2(3 \times)$ und/oder $\mathrm{m}(3 \times)$ | $\mathrm{a}, \mathrm{b}, \mathrm{c}$ |
| Trigonal | $3(1 \times)$ | $\mathrm{c}, \mathrm{a},[210]$ |
| Tetragonal | $4(1 \times)$ | $\mathrm{c}, \mathrm{a},[110]$ |
| Hexagonal | 6 | $\mathrm{c}, \mathrm{a},[210]$ |
| Kubisch | $3(4 \times)$ | $\mathrm{a},[111],[110]$ |

# 1. Basic crystallography Exercise: restrictions of the crystal systems 

1 In tetragonal crystals, the restriction $\alpha=\beta=\gamma=90^{\circ}$ can Right Wrong not be violated

2 The characteristic symmetry element of cubic crystals is Right Wrong the fourfold axis

3 For monoclinic crystals $\beta$ must be $\neq 90^{\circ}$
Right Wrong
$4 \alpha=\beta=y=90^{\circ}$ is not possible for triclinic crystals
Right Wrong

5 A crystal system with $\alpha=\beta=\gamma=90^{\circ}$ and $a=b=c$ must
Right
Wrong be denominated "cubic"

6 A crystal system with $\alpha=\beta=\gamma=90^{\circ}$ and $a \neq b \neq c$ can be Right

Wrong denominated "orthorhombic"

# 1. Basic crystallography <br> Crystal Classes- exercises 

Determine the SE of following arrangements
Specify the crystal system and the crystal class


# 1. Basic crystallography Space groups: Introduction 

A crystallographic space group is a set of symmetry operations which describes all periodic patterns (230, 3D) in 3D space

Space group notations (H.-M.-notation)

- first letter: Bravais-type (lattice centering)
- second and subsequent numbers and letters: symmetry elements along characteristic directions
- caution: some notations are reduced


## P 3121

# 1. Basic crystallography Space group tables 

reciprocal space
Generators selected (1); $t(1,0,0) ; t(0,1,0) ; t(0,0,1) ;(2) ;(4)$

## Positions

Multiplicity,
Coordinates

## Wyckoff letter,

Site symmety
(1) $x, y, z$
(2) $\bar{y}, x-y, z+\frac{1}{3}$
(5) $x-y, \bar{y}, \bar{z}+\frac{2}{3}$
(3) $\bar{x}+y, \bar{x}, z+\frac{2}{3}$
(6) $\bar{x}, \bar{x}+y, \bar{z}+\frac{1}{3}$
$, x, \frac{1}{6} \quad \bar{x}, \bar{x}, \frac{1}{2}$
$3 a \quad 2$
$0, x, \frac{1}{6}$
$\bar{x}, \bar{x}, \frac{1}{2}$
$\bar{x}, \bar{x}, 0$
Symmatry of special projections Along [001] p31
$\mathbf{a}^{\prime}=\mathbf{a} \quad \mathbf{b}^{\prime}=\mathbf{b}$
Origin at $0,0, z$

## Origin at $x, 0$,

Maximal non-isomorphic subgroups
I
$\begin{cases}{[2] P 3_{1} 11\left(P 3_{1}, 144\right)} & 1 ; 2 ; 3 \\ {[3] P 121(C 2,5)} & 1 ; 4 \\ {[3] P 121(C 2,5)} & 1 ; 5 \\ {[3] P 121(C 2,5)} & 1 ; 6\end{cases}$

IIa none
IIb $[3] H 3,21\left(\mathbf{a}^{\prime}=3 \mathbf{a}, \mathbf{b}^{\prime}=3 \mathbf{b}\right)(P 3,12,151)$

## Maximal isomorphic subgroups of lowest index

IIc $\quad[2] P 3_{2} 21\left(\mathbf{c}^{\prime}=2 \mathbf{c}\right)(154) ;[4] P 3_{1} 21\left(\mathbf{a}^{\prime}=2 \mathbf{a}, \mathbf{b}^{\prime}=2 \mathbf{b}\right)(152) ;[7] P 3,21\left(\mathbf{c}^{\prime}=7 \mathbf{c}\right)(152)$

## Minimal non-isomorphic supergroups

I $\quad[2] P 6,22(178) ;[2] P 6422(181)$
II [3] H3,21(P3,12,151); [3] R32 (obverse) (155); [3] R32 (reverse) (155); [3] P321 (c $\left.=\frac{1}{3} \mathbf{c}\right)(150)$

Example 1: Pm (full notation)

- P means primitive (no centering)
- characteristic SE: $\mathbf{m} \rightarrow$ monoclinic
- m perpendicular to [010]

Example 2: $\mathbf{C 2} / \mathrm{c}$ (full notation)

- C means ab-plane (001) is centered
- characteristic SE: 2 or $\mathbf{c} \rightarrow$ monoclinic.
- 2 along $b, c$ perpendicular to $b$


## Example 3: $P 6_{3} / m m c=P 6_{3} / m 2 / m 2 / c$ (full notation)

- $\mathbf{P}$ means primitive (no centering)
- characteristic SE: $6_{3} \rightarrow$ hexagonal
- $6_{3}$ along $\mathrm{c}, \mathrm{m}$ perpendicular $\mathrm{c}, 2$ along $\mathrm{a}, \mathrm{m}$ perpendicular a , 2 along [210], c perpendicular [210]

Information derived from H.M. notation

- Crystal system, Crystal class
- Centrosymmetric / non centrosymmetric
- Reflection conditions ("Extinctions")


## 2. Basic diffraction physics Model for X-ray diffraction (XRD)

## Scattering can be separated into:

- scattering by all electrons of the distinct atoms of the structure (atomic form factor)
- scattering by all atoms of the structure (structure factor, convolution lattice and unit cell)
- scattering by the whole crystal (finite size effects: broadening of FT)

4 atoms



8 atoms

## Mathematical description: Fourier transformation

Object (crystal) real space electron density $\rho(\mathbf{r})$

diffraction pattern reciprocal space structure factor $\mathrm{F}\left(\mathrm{r}^{*}\right)$

## 2. Basic diffraction physics

Example: FFTs in electron microscopy


## 1 nm



## Basic FTs in XRD I: Atomic scattering factor

$$
\rho(r)=\int_{V^{*}} F\left(r^{*}\right) \exp \left(-2 i \pi\left(r r^{*}\right)\right) d r^{*} \quad F\left(r^{*}\right)=\int_{V} \rho(r) \exp \left(2 i \pi\left(r r^{*}\right)\right) d r
$$

- Scattering by atoms (FT of $\mathrm{U}_{\mathrm{a}}(\mathrm{r})$ )

$$
f_{a}\left(r^{*}\right)=\int_{V} U_{a}(r)\left(\sin \left(2 \pi r r^{*}\right)\right) / 2 \pi r r^{*} d r \sim \sum f_{e l} \cdot\left(r^{*}\right) \text {, with: } U_{a}(r)=4 \pi r^{2} \rho_{a}(r)
$$

## Experimental consequences

- high diffracted intensity at low $\theta$, e.g. left side of powder DP
- "light atoms problem", consequence: alternative methods



## 2. Basic diffraction physics

## Basic FTs in XRD II: periodic and infinite crystals

$$
\rho(r)=\int_{V^{*}} F\left(r^{*}\right) \exp \left(-2 i \pi\left(r r^{*}\right)\right) d r^{*} \quad F\left(r^{*}\right)=\int_{V} \rho(r) \exp \left(2 i \pi\left(r r^{*}\right)\right) d r
$$

- $\rho(r)$ exhibits the periodicity of the crystal: Fourier series $\rho(r)=\sum C_{g} \exp \left(2 i \pi\left(r^{*} r\right)\right), C_{g}$ : Fourier coefficient
$=1 / V \sum F_{h k l} \exp (2 i \pi(h x+k y+\mid z))$, hkl: rec. space, xyz: dir. space
$F_{h k l}=\sum_{f_{i}} \exp (2 \pi i(h x+k y+l z))$, summing up the contributions of all atoms


Example: graph of a rectangular $\mathrm{f}(\mathrm{x})=\boldsymbol{\operatorname { s i n }} \mathrm{x}+\sin 3 \mathrm{x} / 3+\sin 5 \mathrm{x} / 5+\sin 7 \mathrm{x} / 7+\ldots$

## 2. Basic diffraction physics Scattered intensity: Structure factor $F_{\text {hkl }}$

$$
\begin{aligned}
& F_{h k l}=\Sigma f_{i} \exp (2 \pi i(h x+k y+l z)) \\
& =\Sigma f_{i}[\cos (2 \pi(h x+k y+i z))+i \sin (2 \pi(h x+k y+I z))] \\
& =\Sigma A_{i}+i B_{i}
\end{aligned}
$$

Remarks

- $\mathrm{F}_{\mathrm{hk}}$ : summing up the contributions from all atoms
- "All structural information is in one reflection"
- $\phi$ : phase of $\mathrm{F}_{\mathrm{hkl}}$ contains structure information, $\phi=\arctan$ B/A
- $I F_{\text {hkl }} I=\left(A^{2}+B^{2}\right)^{1 / 2} \sim I=$ : phase problem, i.e. phase is lost
- Friedel's law: $I_{\text {hkl }}=I_{-h-k-1}$
- Symmetry of DP: centrosymmetric (first approximation)


# 2. Basic diffraction physics Calculations of structure factors: examples 

$$
\begin{aligned}
& F_{h k l}=\sum f_{i} \exp (2 \pi i(h x+k y+l z)) \\
& =\sum f_{i}[\cos (2 \pi(h x+k y+l z))+i \sin (2 \pi(h x+k y+l z))]
\end{aligned}
$$

- Primitive (one atom type)
- Calculation for CsCl
- BCC (one atom type)
- Calculation for NaCl


## 2. Basic diffraction physics Example NaCl

Each peak corresponds to one set of hkl planes (or equivalent plane)


# 2. Basic diffraction physics NaCl vs. KCl 




## 2. Basic diffraction physics <br> Geometrical approach, Bragg's law (BL)

Description by wave vector: $k_{1}$ : incident beam, $k_{D}$ : diffracted beam; $\left|k_{I}\right|=\left|k_{D}\right|=1 / \lambda$


Constructive Interference


Destructive Interference


$$
\begin{aligned}
& A C+C D=n \lambda=2 d \sin \theta_{B} \\
& 2 \sin \theta_{B} / \lambda=n / d=n l d^{*} l
\end{aligned}
$$

# 2. Basic diffraction physics hkl: indices of planes in direct space 

$"(h k l)=(110) "$
Procedure in three steps


1. Select 000
2. Mark intercept of the plane, i. e. reciprocal values $1 / \mathrm{h}$ on $\mathrm{a}, 1 / \mathrm{k}$ on $\mathrm{b}, 1 / \mathrm{l}$ on c

Three points, I = 0 means: plane ||c

3. Draw plane

Convention: right-handed coordinate system
2. Basic diffraction physics hkl planes: examples
(112)

(110)


## 2. Basic diffraction physics Properties / Applications of $\mathrm{d} / \mathrm{d}$ *

- $\mathbf{d} \sim$ to the normal vector of an hkl plane
- IdI ~ distance of two individual hkl planes of the same type

Square form of BL (e.g. orthorhombic)
$-1 / d^{2}=h^{2} / a^{2}+k^{2} / b^{2}+l^{2} / c^{2}$

- $\sin ^{2} \theta=\lambda^{2} / 4\left(h^{2} / a^{2}+k^{2} / b^{2}+I^{2} / c^{2}\right)$
- Application: indexing of DP


## Example for manual indexing

- (e.g. cubic): $(1 / d)^{2}=(1 / a)^{2}\left(h^{2}+k^{2}+l^{2}\right), d=\lambda /(2 \sin \theta)$
- Determine d-spacing of each peak from its $2 \theta$ value (using Bragg's Law)
- Create a table of $1 / \mathbf{d}^{2}$ values for each peak
- Look for a common factor ( $1 / \mathrm{a}^{2}$ ) that can be divided into each of the $(1 / \mathrm{d})^{2}$ values


## 2. Basic diffraction physics Manual indexation: example 1

| 2-theta | d | $1000 / \mathrm{d}^{2}$ |  | hkl |
| :--- | :--- | :--- | :--- | :--- |
| 22.21 | 4.000 | 62.5 | $62.5 / 62.5=1$ | 100 |
| 31.61 | 2.828 | 125.0 | $125.0 / 62.5=2$ | 110 |
| 38.97 | 2.309 | 187.6 | $187.6 / 62.5=3$ | 111 |
| 45.31 | 2.000 | 250.0 | $250.0 / 62.5=4$ | 200 |
| 51.01 | 1.789 | 312.4 | $312.4 / 62.5=5$ | 210 |
| 56.29 | 1.633 | 375.0 | $375.0 / 62.5=6$ | 211 |
| 66.00 | 1.414 | 500.2 | $500.2 / 62.5=8$ | 220 |
| 70.58 | 1.333 | 562.8 | $562.8 / 62.5=9$ | 221 |
| 75.03 | 1.265 | 624.9 | $624.9 / 62.5=10$ | 310 |

## 2. Basic diffraction physics Manual indexation: example 2 (extinctions)

| 2-theta | d | $1000 / \mathrm{d} 2$ |  | hkl |
| :--- | :--- | :--- | :---: | :--- |
| 28.077 | 3.175 | 99.2 | $99.2 / 33=3$ | 111 |
| 32.533 | 2.750 | 132.2 | $132.2 / 33=4$ | 200 |
| 46.672 | 1.945 | 264.3 | $264.3 / 33=8$ | 220 |
| 55.355 | 1.658 | 363.8 | $363.8 / 33=11$ | 311 |
| 58.045 | 1.588 | 396.6 | $396.6 / 33=12$ | 222 |
| 68.140 | 1.375 | 528.9 | $528.9 / 33=16$ | 400 |
| 75.247 | 1.262 | 627.9 | $627.9 / 33=19$ | 331 |
| 77.559 | 1.230 | 661.0 | $661.0 / 33=20$ | 420 |

Systematic of extinctions (general reflection conditions): translation

- Integral reflection conditions: unit cell translations (centers)
-Zonal reflection conditions: glide planes
- Serial reflection conditions: screw axes


## 2. Basic diffraction physics Extinctions: examples

## P2



## 3. Experimental methods Generation of X-rays



## Atomic scale scenario:

- inner shell electrons are striked out
- outer shell electrons fill hole
- production of X-rays due to energy difference between inner and outer shell electron


X-ray tube

## 3. Determination of 3D structures Practical work- Essentials

1. Selection of "good" samples:

- Single crystals: diameter < 0.2 mm
- Homogeneous powders: small crystals of one compound

2. Determination of symmetry and unit cell

3. Measurement of diffracted intensities (automatic procedure)
4. Calculation of possible atomic parameters (structure solution)
5. Refinement of the structure model (PC)

- Comparison of experimental and calculated data
- Atomic parameters are optimized until refinement converges

6. Interpretation of the resulting refinement results

- R-values, convergence, thermal parameters

7. Interpretation of the resulting structure (does it make sense?)

- Interatomic distances, occupancy factors (reasonable values?)
- lonic compounds: compensation of all charges


## 3. Determination of 3D structures Results of diffraction studies- Overview

a) Position of the reflections (Bragg's law): Lattice parameters $(1 / d)^{2}=(1 / a)^{2}\left[h^{2}+k^{2}+I^{2}\right]$
b) Intensity of reflections

- Symmetry of the structure: Space group
- Structure (fractional coordinates)
c) Profile of the reflections
- Crystal size and perfection cf. Scherrer formula: $\Delta(2 \theta)=\lambda / L \cos \theta$
- Indications for structural disorder


