X-ray analysis
1. Basic crystallography
2. Basic diffraction physics
3. Experimental methods

#### Introduction Noble prizes associated with X-ray diffraction

- 1901 W. C. Roentgen (Physics) for the discovery of X-rays.
- 1914 M. von Laue (Physics) for X-ray diffraction from crystals.
- 1915 W. H. and W. L. Bragg (Physics) for structure derived from X-ray diffraction.
- 1917 C. G. Barkla (Physics) for characteristic radiation of elements.
- 1924 K. M. G. Siegbahn (Physics) for X-ray spectroscopy.
- 1927 A. H. Compton (Physics) for scattering of X-rays by electrons.
- 1936 P. Debye (Chemistry) for diffraction of X-rays and electrons in gases.
- 1962 M. Perutz and J. Kendrew (Chemistry) for the structure of hemoglobin.
- 1962 J. Watson, M. Wilkins, and F. Crick (Medicine) for the structure of DNA.
- 1979 A. Cormack and G. Newbold Hounsfield (Medicine) for computed axial tomography.
- 1981 K. M. Siegbahn (Physics) for high resolution electron spectroscopy.
- 1985 H. Hauptman and J. Karle (Chemistry) for direct methods to determine structures.
- 1988 J. Deisenhofer, R. Huber, and H. Michel (Chemistry) for the structures of proteins that are crucial to photosynthesis.



### Introduction What's the result of X-ray analyses?

#### **Crystal data**

Formula sum Crystal system Space group Unit cell dimensions Z  $\begin{array}{l} {\rm Mg_2SiO_4~(Olivine)} \\ {\rm orthorhombic} \\ {\it P~b~n~m~(no.~62)} \\ {\it a=4.75(2)~{\rm \AA},~b=10.25(4)~{\rm \AA},~c=6.00(2)~{\rm \AA}} \\ {\it 4} \end{array}$ 

#### **Atomic coordinates**

Atom	Ox.	Wyck.	X	У	Z
Mg1	+2	4 <i>a</i>	0.00000	0.00000	0.00000
Mg2	+2	4 <i>c</i>	0.00995(600)	0.27734(600)	0.75000
Si1	+4	4 <i>c</i>	0.07373(500)	0.4043(50)	0.25000
01	-2	4 <i>c</i>	0.23242(1000)	0.0918(100)	0.75000
02	-2	4 <i>c</i>	0.2793(100)	0.05078(1000)	0.25000
O3	-2	8 <i>d</i>	0.22266(1000)	0.33594(1000)	0.46289(1000)

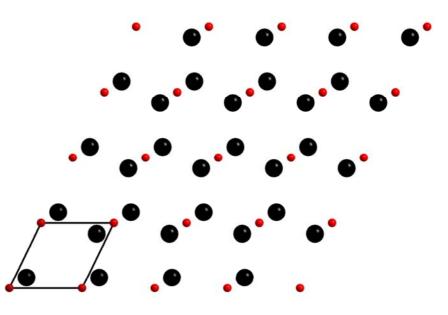
#### Structure

Chemical information (bonding, composition)

Real structure (defects)

### 1. Basic crystallography Lattice, motif and structure

Example: structure and lattice in 2D



#### Lattice

- pattern of points
- no chemical information, mathematical description
- no atoms, but points and lattice vectors (a, b, c,  $\alpha$ ,  $\beta$ ,  $\gamma$ ), unit cell
- Motif (characteristic structural feature, atom, group of atoms...)
- Structure = Lattice + Motif
  - contains chemical information (e.g. environment, bond length...)
  - describes the arrangement of atoms (symmetry of the crystal)

### 1. Basic crystallography Unit cell: interconnection of lattice and structure

#### **Definition:**

Unit cell = parallel sided region of the lattice from which the entire crystal can be constructed by purely translational displacements

- contents of unit cell represents chemical composition (multiples of chemical formula)
- primitive cell: simplest cell, contains one lattice point
- centered cell: more than one point inside unit cell

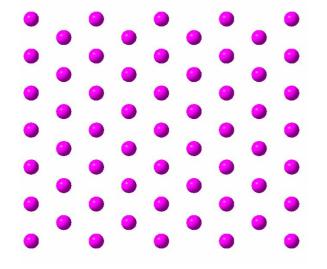
# on nt

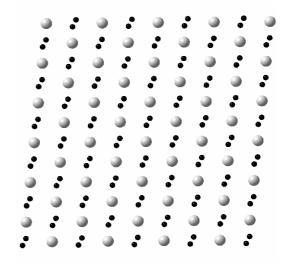
#### **Conventions:**

- 1. Cell edges should coincide with symmetry axes or reflection planes
- 2. The smallest possible cell which fulfills 1. should be chosen

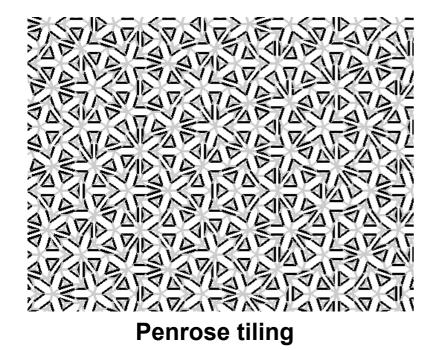
### 1. Basic crystallography Unit cell: exercise

Determine the primitive unit cell and one example for a centered setting

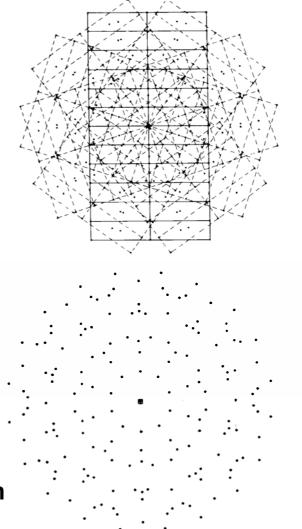




### 1. Basic crystallography Aperiodic structures: e.g. quasicrystals



Forbidden symmetry produced by superposition

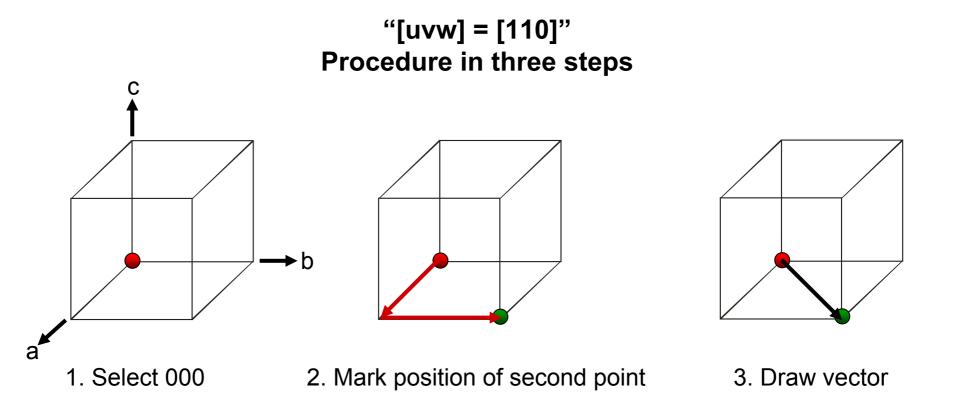


### 1. Basic crystallography Unit cells and crystal system

- millions of periodic structures but 7 types of primitive cells (crystal systems)
- crystal system = particular restriction concerning the unit cell
- crystal system = unit cell with characteristic symmetry elements (later)

Crystal system	<b>Restrictions axes</b>	<b>Restrictions angles</b>
Triclinic	_	-
Monoclinic	-	$\alpha = \gamma = 90^{\circ}$
Orthorhombic	-	$\alpha = \beta = \gamma = 90^{\circ}$
Tetragonal	a = b	$\alpha = \beta = \gamma = 90^{\circ}$
Trigonal	a = b	$\alpha = \beta = 90^{\circ}, \ \gamma = 120^{\circ}$
Hexagonal	a = b	$\alpha = \beta = 90^{\circ}, \ \gamma = 120^{\circ}$
Cubic	a = b = c	$\alpha = \beta = \gamma = 90^{\circ}$

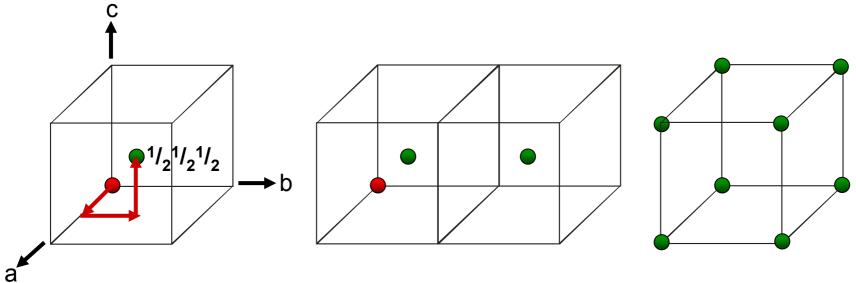
#### 1. Basic crystallography Indexation of directions in direct space



#### **Convention: right-handed coordinate system**

- middle finger: a
- thumb: b
- forefinger: c

### 1. Basic crystallography Fractional coordinates (position of the atoms)

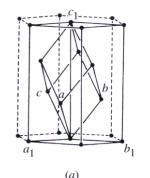


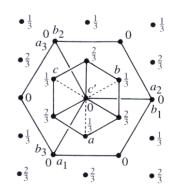
- possible values for x, y, z: [0; 1], atoms are multiplied by translations
- atoms are generated by symmetry elements (later)
- Example: <u>Sphalerite</u> (ZnS)
- Equivalent points are represented by one triplet only
  - equivalent by translation
  - equivalent by other symmetry elements (later)

1. Basic crystallography Centered unit cells- Bravais-type

## The Bravais-types P, F, I, C, A, B, R denote centerings of the unit cells

- Centering of unit cell = Translation
- 7 crystal systems, 14 characteristic unit cells (i.e. Bravais-type)
- 7 types of centerings:
- P: no centering (0,0,0)
- F: translation of each point by (1/2,1/2,0);(0,1/2,1/2);(1/2,0,1/2)
- I: translation of each point by (1/2,1/2,1/2)
- C: translation of each point by (1/2,1/2,0)
- A: translation of each point by (0,1/2,1/2)
- B: translation of each point by (1/2,0,1/2)
- R: translation of each point by (2/3,1/3,1/3);(1/3,2/3,2/3)
- All fractional coordinates are multiplied by centerings





### 1. Basic crystallography Bravais-type: example

<u>Crystal data</u>		
Formula sum	NaCl	
Crystal system	cubic	
Space group	<i>F</i> m -3 m (no. 225)	
Unit cell dimensions	<i>a</i> = 5.6250(5) Å	
Z	4	Halite

#### **Atomic coordinates**

Atom	Ox.	Wyck.	Χ	У	Ζ
Na	+1	4a	0	0	0
CI	-1	4 <i>b</i>	1/2	1/2	1/2

#### 1. Basic crystallography Wyckoff-notation and occupancy factor

#### **Crystal data**

Formula sum Crystal system Space group Unit cell dimensions Z Cu<sub>0.8</sub> In<sub>2.4</sub> Se<sub>4</sub> tetragonal *I*-4 2 m (no. 121) a = 5.7539(3) Å c = 11.519(1) Å 2

#### **Atomic coordinates**

#### **Molecules**

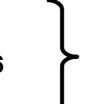
Atom	Ox.	Wyck.	Occ.	X	У	Ζ
Cu1	+1	2a	0.8	0	0	0
In1	+3	4 <i>d</i>	1.0	0	1/2	1/4
In2	+3	2b	0.4	0	0	1/2
Se1	-2	8 <i>i</i>	1.0	1/4	1/4	1/8

#### **Occ.** factors × Wyckoff number = no. of atoms/unit cell

### 1. Basic crystallography Crystallographic symmetry elements

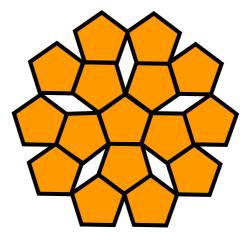
#### SE in crystallography:

- Inversion
- Mirror
- Rotation axes: 1,2,3,4,6
- Translations



**Coupling and combination** 

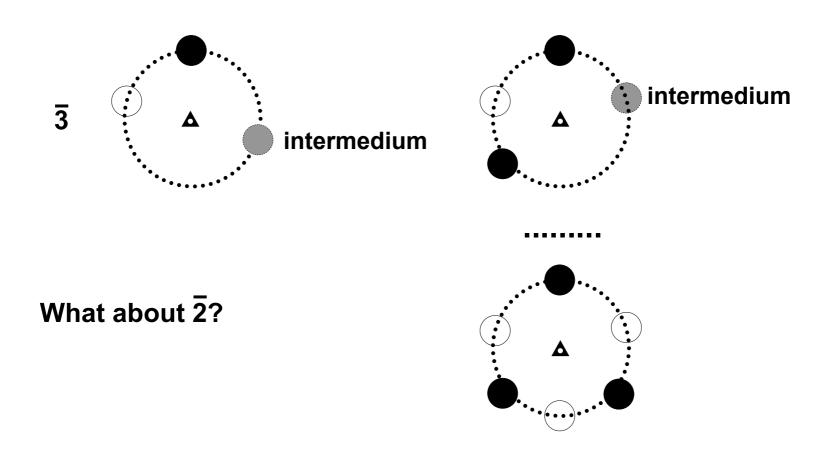
Question: Why is e. g. 5 forbidden in crystal structures?



### 1. Basic crystallography Coupling- rotation and inversions

**Coupling:** 

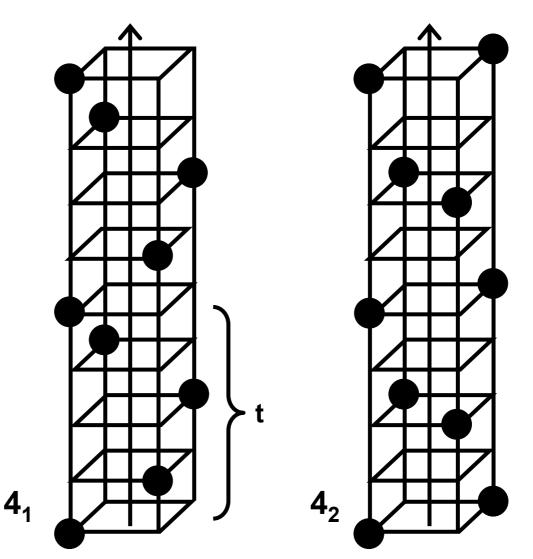
- both SE are applied in one step
- one of the two coupled SE is an intermedium



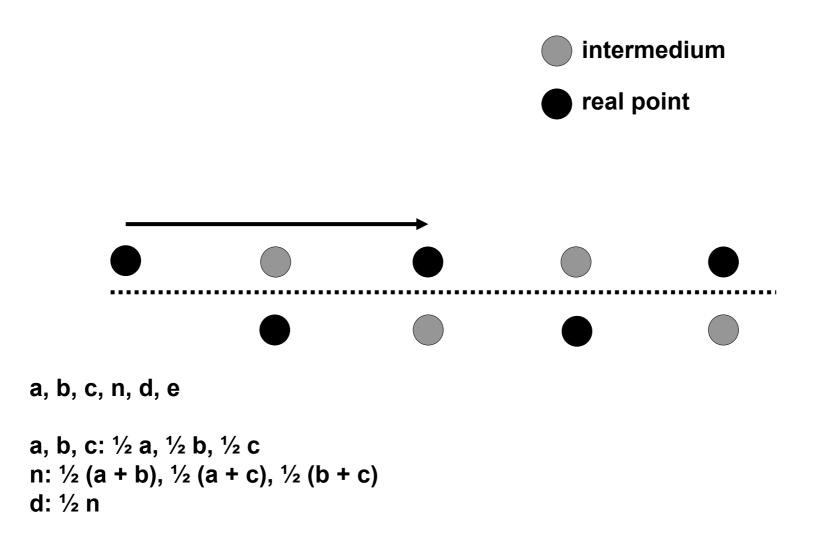
### 1. Basic crystallography Coupling- rotation and translation- Screw axes X<sub>n</sub>

Strategy: X-fold rotation after translation of n/X along the screw axis

 $2_{1} \\ 3_{1} 3_{2} \\ 4_{1} 4_{3} 4_{2} \\ 6_{1} 6_{5} 6_{2} 6_{4} 6_{3}$ 



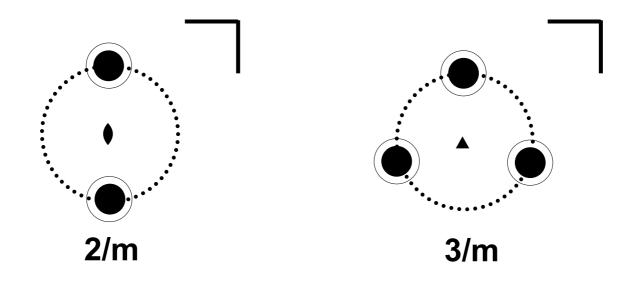
### 1. Basic crystallography Coupling- mirror and translation – Gilde planes



#### 1. Basic crystallography Combination of mirror and rotation

**Combination:** 

- both SE are applied (two steps)
- no intermedium



### 1. Basic crystallography Directions of characteristic symmetry elements

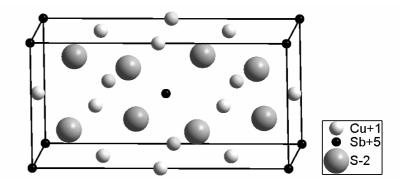
Crystal system	Characteristic SE	Char. direction / sequence
Triclinic	1, 1	-
Monoclinic	2 and/or m	b
Orthorhombic	2 (3×) and/or m(3×)	a, b, c
Trigonal	3 (1×)	c, a, [210]
Tetragonal	4 (1×)	c, a, [110]
Hexagonal	6	c, a, [210]
Cubic	3 (4×)	[111], a, [110]

#### What can we learn from this table?

- The knowledge of the characteristic SE allows to determine the crystal system
- Pseudometrics possible, e. g. monoclinic with  $\beta$  = 90°
- Systematization of SE in space group symbols

### Repetition Basic terms of 3D crystallography

- Lattice, motif and structure
- Unit cell (primitive and centered)
- Wyckoff notation, occupancy factor, composition of crystals
- Crystal system
- Bravais type
- Crystallographic SE



- Coupling of SE: rotoinversion, screw axes, glide planes
- Combination of SE
- Characteristic SE of the crystal casses

### 1. Basic crystallography Crystal class

#### 32 Crystal classes: Combination of SE without translations

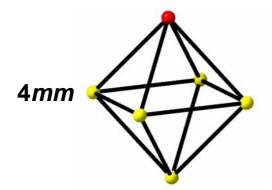
Crystal system	Restriction	Crystal Class
Triclinic	-	1,1
Monoclinic	$- \alpha = \gamma = 90^{\circ}$	2, m, 2/m
Orthorhombic	$\mathbf{a} = \mathbf{b}$ $\alpha = \beta = \gamma = 90^{\circ}$	222, mm2, 2/m 2/m 2/m
Trigonal	$\mathbf{a} = \mathbf{b}$ $\alpha = \beta = 90^{\circ}, \ \gamma = 120^{\circ}$	3, 3, 32, 3m, 32/m
Tetragonal	$\mathbf{a} = \mathbf{b}$ $\alpha = \beta = \gamma = 90^{\circ}$	4, 4, 4/m, 422, 4mm, 42m, 4/m 2/m 2/m
Hexagonal	$\mathbf{a} = \mathbf{b}$ $\alpha = \beta = 90^{\circ}, \ \gamma = 120^{\circ}$	6, <del>6</del> , 6/m, 622, 6mm, <del>6</del> m2, 6/m 2/m 2/m
Cubic	$\mathbf{a} = \mathbf{b} = \mathbf{c}$ $\alpha = \beta = \gamma = 90^{\circ}$	23, 2/m 3, 432, 43m, 4/m 3 2/m

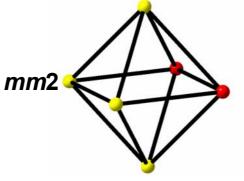
The atoms (A, B and C) of a tetragonal structure are located on:  $0^{1}/_{2}0$  (A),  $1^{1}/_{2}0$  (B) and 000 (C). Assume low symmetry

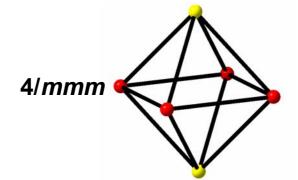
- Sketch the unit cell along [001]
- Determine the composition of a crystal with this structure
- Describe the environment of A, B and C

**Repeat the exercise for a cubic structure!** 

### 1. Basic crystallography Crystal Classes- examples







<b>Crystal System</b>	Charact. SE	<b>Direction / Sequence</b>
Triklin	1, 1	-
Monoklin	2 and/or m	b
Orthorhombisch	2 (3×) und/oder m(3×)	a, b, c
Trigonal	3 (1×)	c, a, [210]
Tetragonal	4 (1×)	c, a, [110]
Hexagonal	6	c, a, [210]
Kubisch	3 (4×)	a, [111], [110]

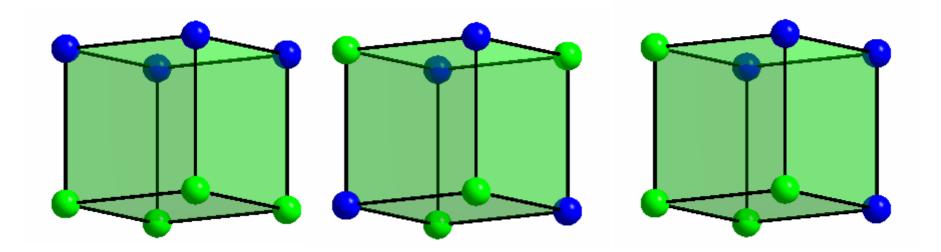
### 1. Basic crystallography Exercise: restrictions of the crystal systems

1	In tetragonal crystals, the restriction $\alpha = \beta = \gamma = 90^{\circ}$ can not be violated	Right	Wrong
2	The characteristic symmetry element of cubic crystals is the fourfold axis	Right	Wrong
3	For monoclinic crystals $\beta$ must be $\neq$ 90°	Right	Wrong
4	$\alpha = \beta = \gamma = 90^{\circ}$ is not possible for triclinic crystals	Right	Wrong
5	A crystal system with $\alpha = \beta = \gamma = 90^{\circ}$ and $a = b = c$ must be denominated "cubic"	Right	Wrong
6	A crystal system with $\alpha = \beta = \gamma = 90^{\circ}$ and $a \neq b \neq c$ can be denominated "orthorhombic"	Right	Wrong

1. Basic crystallography Crystal Classes- exercises

#### **Determine the SE of following arrangements**

#### Specify the crystal system and the crystal class



### 1. Basic crystallography Space groups: Introduction

A crystallographic space group is a set of symmetry operations which describes all periodic patterns (230, 3D) in 3D space

Space group notations (H.-M.-notation)

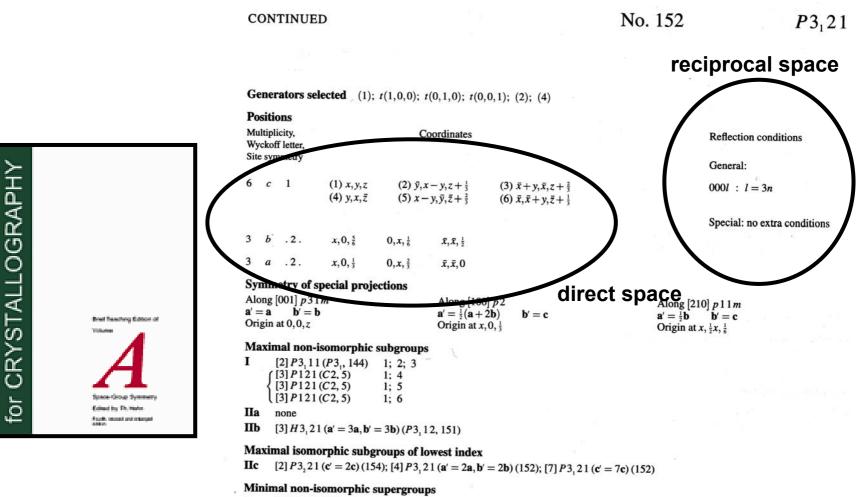
- first letter: Bravais-type (lattice centering)
- second and subsequent numbers and letters: symmetry elements along characteristic directions
- caution: some notations are reduced

*P* 3<sub>1</sub> 2 1

### 1. Basic crystallography Space group tables

TABLES

NTERNATIONAL



- I [2] P6, 22 (178); [2] P6, 22 (181)
- II [3]  $H_{3,21}(P_{3,12,151});$  [3]  $R_{32}$  (obverse) (155); [3]  $R_{32}$  (reverse) (155); [3]  $P_{321}(c' = \frac{1}{3}c)$  (150)

### 1. Basic crystallography Space group: Examples (H.M. notation)

#### Example 1: *Pm* (full notation)

- P means primitive (no centering)
- characteristic SE:  $m \rightarrow monoclinic$
- m perpendicular to [010]

#### Example 2: C2/c (full notation)

- C means ab-plane (001) is centered
- characteristic SE: 2 or  $c \rightarrow$  monoclinic.
- 2 along b, c perpendicular to b

#### Example 3: $P6_3/mmc = P6_3/m 2/m 2/c$ (full notation)

- P means primitive (no centering)
- characteristic SE:  $6_3 \rightarrow$  hexagonal
- 6<sub>3</sub> along c, m perpendicular c, 2 along a, m perpendicular a, 2 along [210], c perpendicular [210]

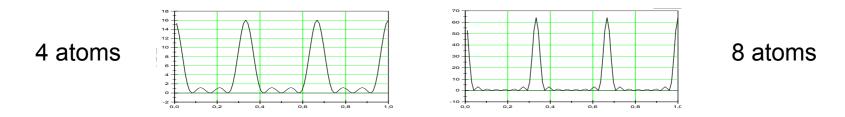
#### Information derived from H.M. notation

- Crystal system, Crystal class
- Centrosymmetric / non centrosymmetric
- Reflection conditions ("Extinctions")

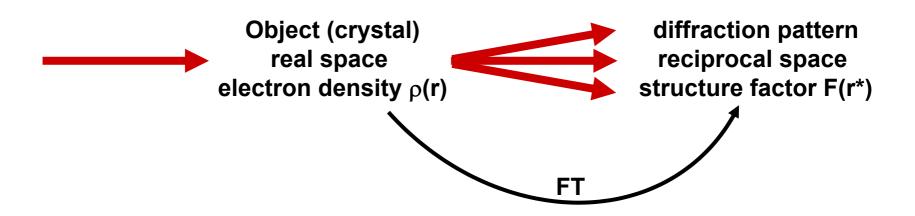
### 2. Basic diffraction physics Model for X-ray diffraction (XRD)

#### Scattering can be separated into:

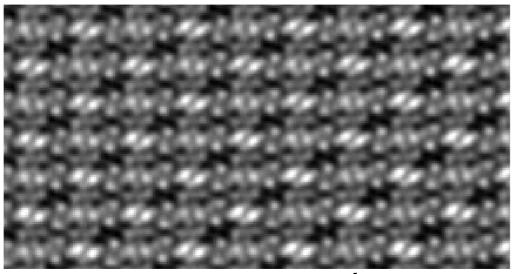
- scattering by all electrons of the distinct atoms of the structure (atomic form factor)
- scattering by all atoms of the structure (structure factor, convolution lattice and unit cell)
- scattering by the whole crystal (finite size effects: broadening of FT)



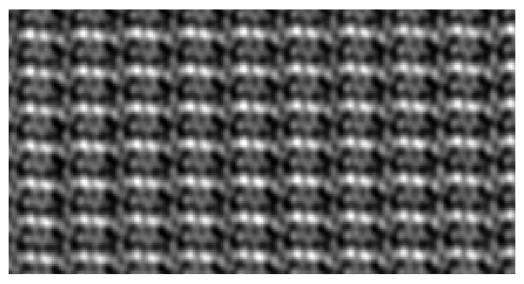
#### **Mathematical description: Fourier transformation**

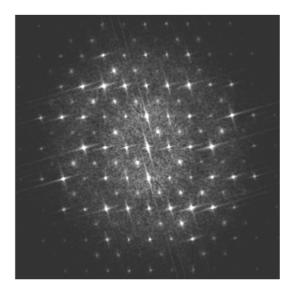


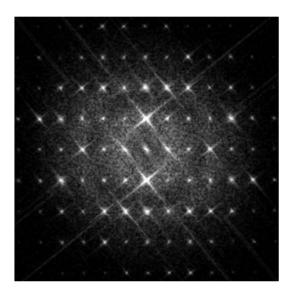
### 2. Basic diffraction physics Example: FFTs in electron microscopy



<u>1 nm</u>







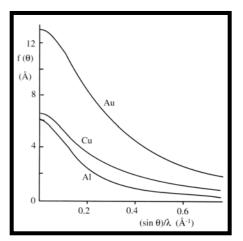
### 2. Basic diffraction physics Basic FTs in XRD I: Atomic scattering factor

$$\rho(\mathbf{r}) = \int_{V^*} \mathbf{F}(\mathbf{r}^*) \exp(-2i\pi(\mathbf{r}\mathbf{r}^*)) d\mathbf{r}^*$$
  $\mathbf{F}(\mathbf{r}^*) = \int_{V} \rho(\mathbf{r}) \exp(2i\pi(\mathbf{r}\mathbf{r}^*)) d\mathbf{r}^*$ 

• Scattering by atoms (FT of U<sub>a</sub>(r))  $f_a(r^*) = \int_V U_a(r) (\sin(2\pi r r^*))/2\pi r r^* dr \sim \sum f_{el}(r^*), \text{ with: } U_a(r) = 4\pi r^2 \rho_a(r)$ 

#### **Experimental consequences**

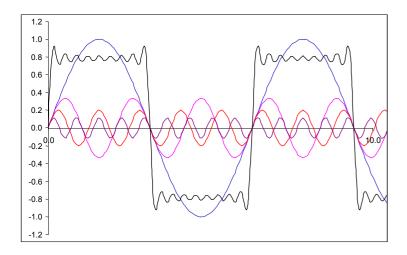
- high diffracted intensity at low  $\theta$ , e.g. left side of powder DP
- "light atoms problem", consequence: alternative methods



### 2. Basic diffraction physics Basic FTs in XRD II: periodic and infinite crystals

$$\rho(\mathbf{r}) = \int_{V^*} \mathbf{F}(\mathbf{r}^*) \exp(-2i\pi(\mathbf{r}\mathbf{r}^*)) d\mathbf{r}^*$$
  $\mathbf{F}(\mathbf{r}^*) = \int_{V} \rho(\mathbf{r}) \exp(2i\pi(\mathbf{r}\mathbf{r}^*)) d\mathbf{r}$ 

- $\rho(\mathbf{r})$  exhibits the periodicity of the crystal: Fourier series  $\rho(\mathbf{r}) = \sum C_g \exp (2i\pi(\mathbf{r}^*\mathbf{r})), C_g$ : Fourier coefficient  $= 1/V \sum F_{hkl} \exp (2i\pi(hx + ky + lz)), hkl$ : rec. space, xyz: dir. space
- $F_{hkl} = \sum f_i \exp(2\pi i(hx + ky + lz))$ , summing up the contributions of all atoms



Example: graph of a rectangular

 $f(x) = \sin x + \sin 3x/3 + \sin 5x/5 + \sin 7x/7 + \dots$ 

### 2. Basic diffraction physics Scattered intensity: Structure factor F<sub>hkl</sub>

$$F_{hkl} = \Sigma f_i \exp(2\pi i(hx + ky + lz))$$
  

$$= \Sigma f_i [\cos (2\pi (hx + ky + lz)) + i \sin (2\pi (hx + ky + lz))]$$
  

$$= \Sigma A_i + i B_i$$
  
imaginary  

#### Remarks

- $F_{hkl}$ : summing up the contributions from all atoms
- "All structural information is in one reflection"
- $\phi$ : phase of F<sub>hkl</sub> contains structure information,  $\phi$  = arctan B/A
- $IF_{hkl}I = (A^2 + B^2)^{1/2} \sim I = :$  phase problem, i.e. phase is lost
- Friedel's law: I<sub>hkl</sub> = I<sub>-h-k-l</sub>
- Symmetry of DP: centrosymmetric (first approximation)

### 2. Basic diffraction physics Calculations of structure factors: examples

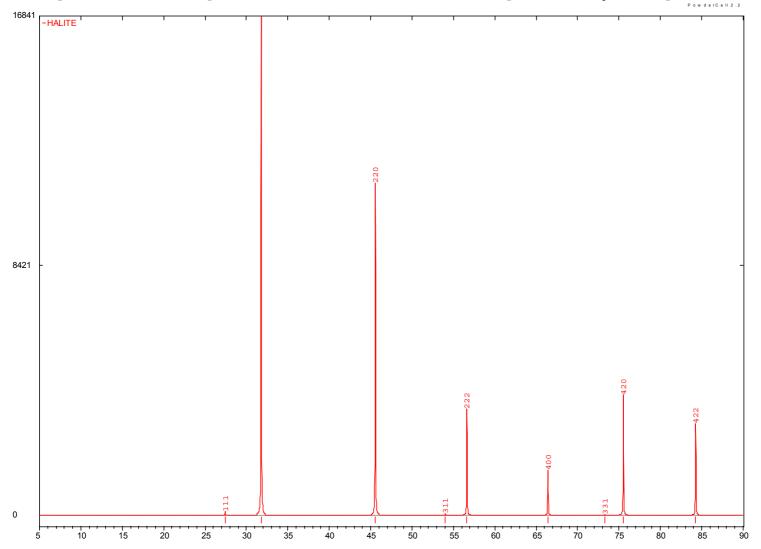
$$\mathbf{F}_{hkl} = \sum \mathbf{f}_i \exp(2\pi \mathbf{i}(h\mathbf{x} + k\mathbf{y} + l\mathbf{z}))$$

= 
$$\sum f_i [\cos (2\pi(hx + ky + lz)) + i \sin (2\pi(hx + ky + lz))]$$

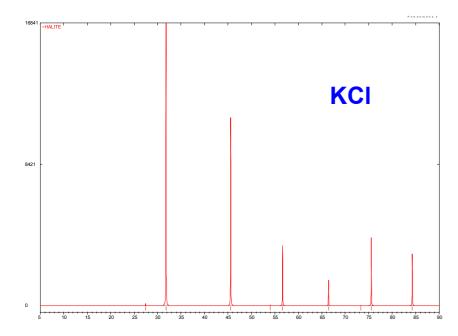
- Primitive (one atom type)
- Calculation for CsCl
- BCC (one atom type)
- Calculation for NaCl

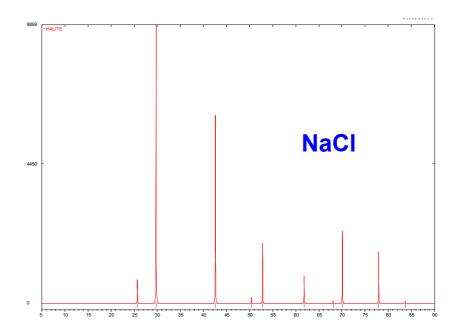
### 2. Basic diffraction physics Example NaCl

Each peak corresponds to one set of hkl planes (or equivalent plane)



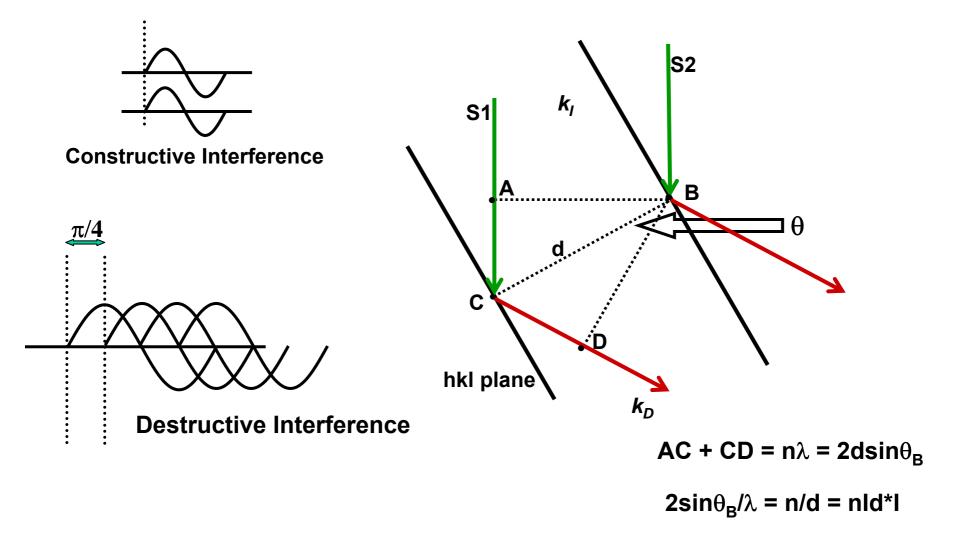
### 2. Basic diffraction physics NaCl vs. KCl



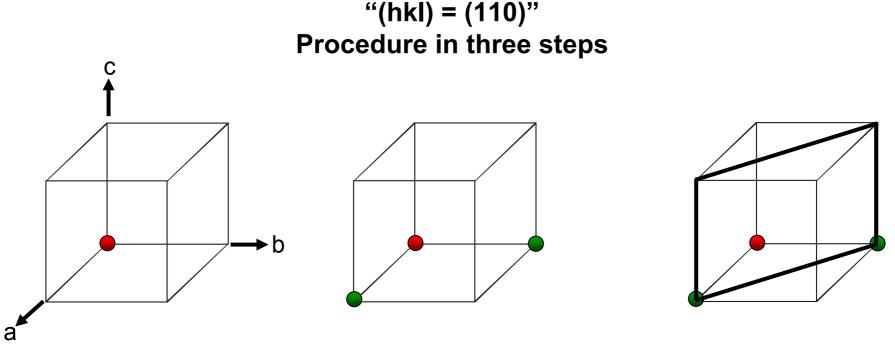


### 2. Basic diffraction physics Geometrical approach, Bragg's law (BL)

Description by wave vector:  $k_I$ : incident beam,  $k_D$ : diffracted beam;  $lk_I l = lk_D l = 1/\lambda$ 



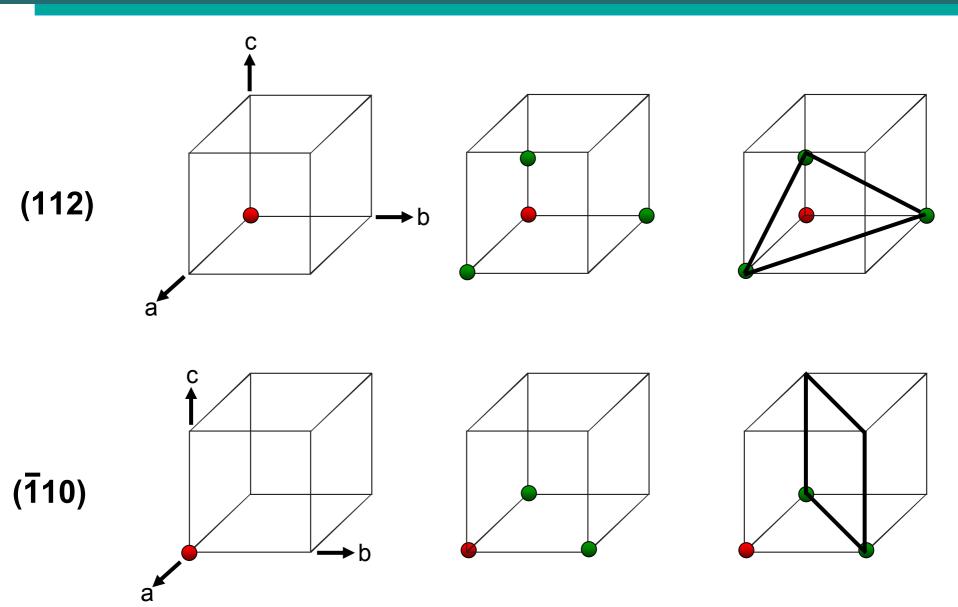
### 2. Basic diffraction physics hkl: indices of planes in direct space



- 1. Select 000 2. Mark intercept of the plane, i. e. reciprocal values 1/h on a, 1/k on b, 1/l on c Three points, I = 0 means: plane || c
- 3. Draw plane

#### **Convention: right-handed coordinate system**

### 2. Basic diffraction physics hkl planes: examples



### 2. Basic diffraction physics Properties / Applications of d/d\*

- d ~ to the normal vector of an hkl plane
- IdI ~ distance of two individual hkl planes of the same type

#### Square form of BL (e.g. orthorhombic)

- $1/d^2 = h^2/a^2 + k^2/b^2 + l^2/c^2$
- $\sin^2\theta = \lambda^2/4 (h^2/a^2 + k^2/b^2 + l^2/c^2)$
- Application: indexing of DP

#### **Example for manual indexing**

- (e.g. cubic):  $(1/d)^2 = (1/a)^2 (h^2 + k^2 + l^2)$ ,  $d = \lambda/(2\sin\theta)$
- Determine d-spacing of each peak from its 2θ value (using Bragg's Law)
- Create a table of 1/d<sup>2</sup> values for each peak
- Look for a common factor (1/a<sup>2</sup>) that can be divided into each of the (1/d)<sup>2</sup> values

### 2. Basic diffraction physics Manual indexation: example 1

2-theta	d	1000/d <sup>2</sup>		hkl
22.21	4.000	62.5	62.5/62.5=1	100
31.61	2.828	125.0	125.0/62.5=2	110
38.97	2.309	187.6	187.6/62.5=3	111
45.31	2.000	250.0	250.0/62.5=4	200
51.01	1.789	312.4	312.4/62.5=5	210
56.29	1.633	375.0	375.0/62.5=6	211
66.00	1.414	500.2	500.2/62.5=8	220
70.58	1.333	562.8	562.8/62.5=9	221
75.03	1.265	624.9	624.9/62.5=10	310

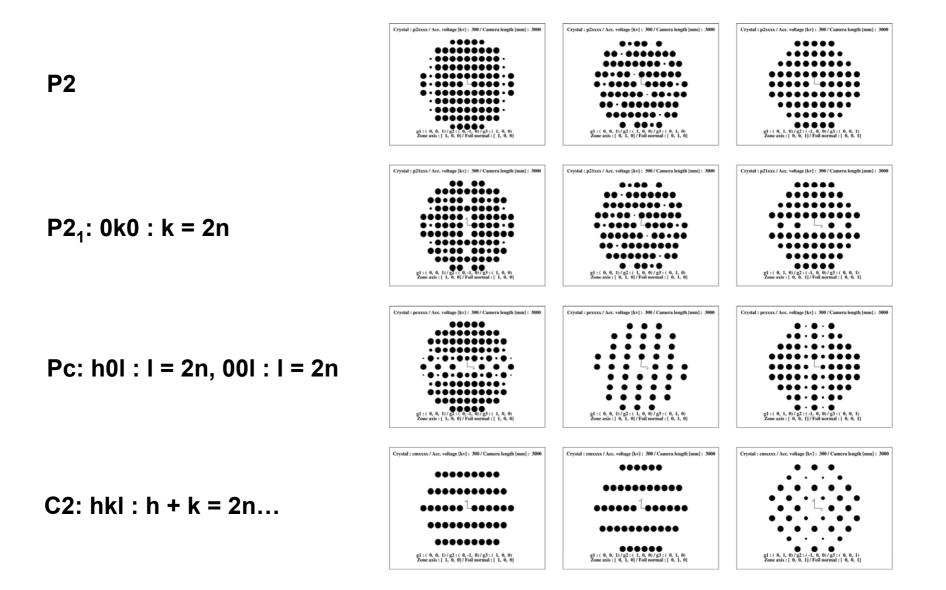
### 2. Basic diffraction physics Manual indexation: example 2 (extinctions)

2-theta	d	1000/d	2	hkl
28.077	3.175	99.2	99.2/33=3	111
32.533	2.750	132.2	132.2/33=4	200
46.672	1.945	264.3	264.3/33=8	220
55.355	1.658	363.8	363.8/33=11	311
58.045	1.588	396.6	396.6/33=12	222
68.140	1.375	528.9	528.9/33=16	400
75.247	1.262	627.9	627.9/33=19	331
77.559	1.230	661.0	661.0/33=20	420

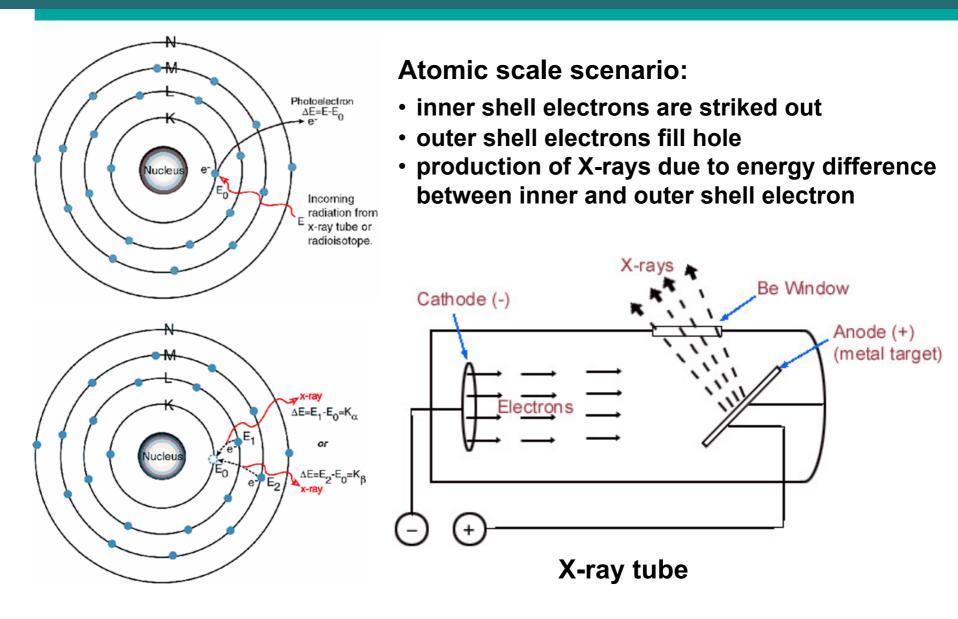
Systematic of extinctions (general reflection conditions): translation

- Integral reflection conditions: unit cell translations (centers)
- Zonal reflection conditions: glide planes
- Serial reflection conditions: screw axes

#### 2. Basic diffraction physics Extinctions: examples



### 3. Experimental methods Generation of X-rays



### 3. Determination of 3D structures Practical work- Essentials

- 1. Selection of "good" samples:
- Single crystals: diameter < 0.2 mm
- Homogeneous powders: small crystals of one compound
- 2. Determination of symmetry and unit cell
- 3. Measurement of diffracted intensities (automatic procedure)
- 4. Calculation of possible atomic parameters (structure solution)
- 5. Refinement of the structure model (PC)
- Comparison of experimental and calculated data
- Atomic parameters are optimized until refinement converges
- 6. Interpretation of the resulting refinement results
- R-values, convergence, thermal parameters
- 7. Interpretation of the resulting structure (does it make sense?)
- Interatomic distances, occupancy factors (reasonable values?)
- Ionic compounds: compensation of all charges



### 3. Determination of 3D structures Results of diffraction studies- Overview

a) Position of the reflections (Bragg's law): Lattice parameters  $(1/d)^2 = (1/a)^2 [h^2 + k^2 + l^2]$ 

#### b) Intensity of reflections

- Symmetry of the structure: Space group
- Structure (fractional coordinates)
- c) Profile of the reflections
- Crystal size and perfection
   cf. Scherrer formula: Δ(2θ) = λ/Lcosθ
- Indications for structural disorder

